CH15 – STATISTICS

Exercise 15.1 Page: 360

Find the mean deviation about the mean for the data in Exercises 1 and 2.

1. 4, 7, 8, 9, 10, 12, 13, 17

Solution:-

First, we have to find ($\bar{x}$) of the given data.

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = \frac{80}{8} = 10$$

So, the respective values of the deviations from mean,

i.e., $x_i - \bar{x}$ are, 10 – 4 = 6, 10 – 7 = 3, 10 – 8 = 2, 10 – 9 = 1, 10 – 10 = 0,
10 – 12 = – 2, 10 – 13 = – 3, 10 – 17 = – 7

6, 3, 2, 1, 0, -2, -3, -7

Now, absolute values of the deviations,

6, 3, 2, 1, 0, 2, 3, 7

$\therefore \sum_{i=1}^{8} |x_i - \bar{x}| = 24$

MD = sum of deviations/ number of observations

= 24/8

= 3

So, the mean deviation for the given data is 3.

2. 38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution:-

First, we have to find ($\bar{x}$) of the given data.

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = \frac{500}{10} = 50$$

So, the respective values of the deviations from mean,

i.e., $x_i - \bar{x}$ are, 50 – 38 = -12, 50 -70 = -20, 50 – 48 = 2, 50 – 40 = 10, 50 – 42 = 8,
50 – 55 = – 5, 50 – 63 = – 13, 50 – 46 = 4, 50 – 54 = -4, 50 – 44 = 6

-12, 20, -2, -10, -8, 5, 13, -4, 4, -6

Now, absolute values of the deviations,

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

$\therefore \sum_{i=1}^{10} |x_i - \bar{x}| = 84$

MD = sum of deviations/ number of observations

= 84/10

= 8.4

So, the mean deviation for the given data is 8.4.

Find the mean deviation about the median for the data in Exercises 3 and 4.

3. 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solution:-

First, we have to arrange the given observations into ascending order.

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18.

The number of observations is 12.

Then,

Median = $((12/2)^{th}$ observation + $((12/2)+1)^{th}$ observation)/2

$(12/2)^{th}$ observation = 6^{th} = 13

$(12/2)+1)^{th}$ observation = 6 + 1

= 7^{th} = 14

Median = (13 + 14)/2

= 27/2

= 13.5

So, the absolute values of the respective deviations from the median, i.e.,

$|x_i - M|$ are

3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

$\therefore \sum_{i=1}^{12}|x_i - M| = 28$

Mean Deviation

$M.D.(M) = \frac{1}{12}\sum_{i=1}^{12}|x_i - M|$

= (1/12) × 28

= 2.33

So, the mean deviation about the median for the given data is 2.33.

4. 36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Solution:-

First, we have to arrange the given observations into ascending order.

36, 42, 45, 46, 46, 49, 51, 53, 60, 72.

The number of observations is 10.

Then,

Median = $((10/2)^{th}$ observation + $((10/2)+1)^{th}$ observation)/2

$(10/2)^{th}$ observation = 5^{th} = 46

$(10/2)+1)^{th}$ observation = 5 + 1

= 6^{th} = 49

Median = (46 + 49)/2

= 95
= 47.5
So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are
11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

$\therefore \sum_{i=1}^{10} |x_i - M| = 70$

Mean Deviation

$M.D.(M) = \frac{1}{10}\sum_{i=1}^{10} |x_i - M|$

= (1/10) × 70
= 7
So, the mean deviation about the median for the given data is 7.

Find the mean deviation about the mean for the data in Exercises 5 and 6.
5.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Solution:-
Let us make the table of the given data and append other columns after calculations.

X_i	f_i	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

The sum of calculated data,

$$N = \sum_{i=1}^{5} f_i = 25, \sum_{i=1}^{5} f_i x_i = 350$$

Now, we have to find ($\bar{x}$) by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{25} \times 350 = 14$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

From the table, $\sum_{i=1}^{5} f_i |x_i - \bar{x}| = 158$

Therefore $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i |x_i - \bar{x}|$

= (1/25) × 158

= 6.32

So, the mean deviation about the mean for the given data is 6.32.

6.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Solution:-

Let us make the table of the given data and append other columns after calculations.

Xi	fi	fixi	$\|xi - \bar{x}\|$	fi $\|xi - \bar{x}\|$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320

	80	4000		1280

The sum of calculated data,

$$N = \sum_{i=1}^{5} f_i = 80, \sum_{i=1}^{5} f_i x_i = 4000$$

Now, we have to find ($\bar{x}$) by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{80} \times 4000 = 50$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

From the table, $\sum_{i=1}^{5} f_i |x_i - \bar{x}| = 1280$

Therefore $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i |x_i - \bar{x}|$

= (1/80) × 1280

= 16

So, the mean deviation about the mean for the given data is 16.

Find the mean deviation about the median for the data in Exercises 7 and 8.

7.

xi	5	7	9	10	12	15
fi	8	6	2	2	2	6

Solution:-

Let us make the table of the given data and append other columns after calculations.

Xi	fi	c.f.	|xi – M|	fi |xi – M|
5	8	8	2	16
7	6	14	0	0

9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48

Now, N = 26, which is even.
Median is the mean of the 13th and 14th observations. Both of these observations lie in the cumulative frequency of 14, for which the corresponding observation is 7.
Then,
Median = (13th observation + 14th observation)/2
= (7 + 7)/2
= 14/2
= 7
So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are shown in the table.

Therefore $\sum_{i=1}^{6} f_i = 26$ and $\sum_{i=1}^{6} f_i |x_i - M| = 84$

And $M.D.(M) = \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - M|$

= (1/26) × 84

= 3.23

Hence, the mean deviation about the median for the given data is 3.23.

8.

xi	15	21	27	30	35
fi	3	5	6	7	8

Solution:-
Let us make the table of the given data and append other columns after calculations.

Xi	fi	c.f.	|xi – M|	fi |xi – M|
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40

Now, N = 29, which is odd.
So, 29/2 = 14.5
The cumulative frequency greater than 14.5 is 21, for which the corresponding observation is 30.
Then,
Median = (15^{th} observation + 16^{th} observation)/2
= (30 + 30)/2
= 60/2
= 30
So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are shown in the table.

Therefore $\sum_{i=1}^{5} f_i = 29$ and $\sum_{i=1}^{5} f_i |x_i - M| = 148$

And $M.D.(M) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - M|$

= (1/29) × 148

= 5.1

Hence, the mean deviation about the median for the given data is 5.1.

Find the mean deviation about the mean for the data in Exercises 9 and 10.
9.

Income per day in ₹	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800
Number of persons	4	8	9	10	7	5	4	3

Solution:-
Let us make the table of the given data and append other columns after calculations.

Income per d in ₹	Number of persons fi	Midpoints xi	fixi	|xi – x̄	fi|xi – x̄
0 – 100	4	50	200	308	1232
100 – 200	8	150	1200	208	1664
200 – 300	9	250	2250	108	972
300 – 400	10	350	3500	8	80
400 – 500	7	450	3150	92	644
500 – 600	5	550	2750	192	960
600 – 700	4	650	2600	292	1160
700 – 800	3	750	2250	392	1176
	50		17900		7896

The sum of calculated data,

$$N = \sum_{i=1}^{8} f_i = 50, \sum_{i=1}^{8} f_i x_i = 17900$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{8} f_i x_i = \frac{1}{50} \times 17900 = 358$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

So, $\sum_{i=1}^{8} f_i |x_i - \bar{x}| = 7896$

And $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{8} f_i |x_i - \bar{x}|$

= (1/50) × 7896

= 157.92

Hence, the mean deviation about the mean for the given data is 157.92.

10.

Height in cm	95 – 10	105 – 1	115 – 1	125 – 1	135 – 1	145 – 1
Number of b	9	13	26	30	12	10

Solution:-

Let us make the table of the given data and append other columns after calculations.

Height in c	Number of bo	Midpoints xi	fixi	|xi – x̄|	fi|xi – x̄|
95 – 105	9	100	900	25.3	227.7
105 – 115	13	110	1430	15.3	198.9
115 – 125	26	120	3120	5.3	137.8

125 – 135	30	130	3900	4.7	141
135 – 145	12	140	1680	14.7	176.4
145 – 155	10	150	1500	24.7	247
	100		12530		1128.8

The sum of calculated data,

$$N = \sum_{i=1}^{6} f_i = 100, \sum_{i=1}^{6} f_i x_i = 12530$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i = \frac{1}{100} \times 12530 = 125.3$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

So $\sum_{i=1}^{6} f_i |x_i - \bar{x}| = 1128.8$

And $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \bar{x}|$

= (1/100) ×1128.8

= 11.28

Hence, the mean deviation about the mean for the given data is 11.28.

11. Find the mean deviation about median for the following data.

Marks	0 -10	10 -20	20 – 30	30 – 40	40 – 50	50 – 60
Number of girls	6	8	14	16	4	2

Solution:-

Let us make the table of the given data and append other columns after calculations.

Marks	Number of girls fi	Cumulative frequency (c.f.)	Mid - points xi	- Med|	fi|xi – Med|
0 – 10	6	6	5	22.85	137.1
10 – 20	8	14	15	12.85	102.8
20 – 30	14	28	25	2.85	39.9
30 – 40	16	44	35	7.15	114.4
40 – 50	4	48	45	17.15	68.6
50 – 60	2	50	55	27.15	54.3
	50				517.1

The class interval containing $N^{th}/2$ or 25^{th} item is 20-30.
So, 20-30 is the median class.
Then,
Median = l + (((N/2) – c)/f) × h
Where, l = 20, c = 14, f = 14, h = 10 and n = 50
Median = 20 + (((25 – 14))/14) × 10
= 20 + 7.85
= 27.85

The absolute values of the deviations from the median, i.e., $|x_i - \text{Med}|$, as shown in the table.

So $\sum_{i=1}^{6} f_i|x_i - \text{Med.}| = 517.1$

And $\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - \text{Med.}|$

= (1/50) × 517.1

= 10.34

Hence, the mean deviation about the median for the given data is 10.34.

12. Calculate the mean deviation about median age for the age distribution of 100 persons given below.

Age (in years)	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40	41 – 45	46 – 50	51 – 55
Number	5	6	12	14	26	12	16	9

[Hint: Convert the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval]

Solution:-

The given data is converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding the 0.5 to the upper limit of each class intervals and append other columns after calculations.

Age	Number fi	Cumulative frequency (c.f.)	Midpoints xi	|xi – Med|	fi|xi – Med|
15.5 – 20.5	5	5	18	20	100
20.5 – 25.5	6	11	23	15	90

25.5 – 30.5	12	23	28	10	120
30.5 – 35.5	14	37	33	5	70
35.5 – 40.5	26	63	38	0	0
40.5 – 45.5	12	75	43	5	60
45.5 – 50.5	16	91	48	10	160
50.5 – 55.5	9	100	53	15	135
	100				735

The class interval containing $N^{th}/2$ or 50^{th} item is 35.5 – 40.5
So, 35.5 – 40.5 is the median class.
Then,
Median = l + (((N/2) – c)/f) × h
Where, l = 35.5, c = 37, f = 26, h = 5 and N = 100
Median = 35.5 + (((50 – 37))/26) × 5
= 35.5 + 2.5
= 38

The absolute values of the deviations from the median, i.e., $|x_i - \text{Med}|$, as shown in the table.

So $\sum_{i=1}^{8} f_i |x_i - \text{Med.}| = 735$

And $\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \text{Med.}|$

= (1/100) × 735

= 7.35

Hence, the mean deviation about the median for the given data is 7.35.

Exercise 15.2 Page: 371

Find the mean and variance for each of the data in Exercise 1 to 5.

1. 6, 7, 10, 12, 13, 4, 8, 12

Solution:-

We have,

Mean = $\bar{x} = \frac{\sum_{i=1}^{a} x_i}{n}$

Where, n = number of observation

$\sum_{i=1}^{a} x_i$ = sum of total observation

So, $\bar{x} = (6 + 7 + 10 + 12 + 13 + 4 + 8 + 12)/8$

$= 72/8$

$= 9$

Let us make the table of the given data and append other columns after calculations.

Xi	Deviations from mean (xi – x̄)	(xi – x̄)2
6	6 – 9 = -3	9
7	7 – 9 = -2	4
10	10 – 9 = 1	1
12	12 – 9 = 3	9

13	13 – 9 = 4	16
4	4 – 9 = – 5	25
8	8 – 9 = – 1	1
12	12 – 9 = 3	9
		74

We know that the Variance,

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \bar{x})^2$

$\sigma^2 = (1/8) \times 74$

$= 9.2$

∴Mean = 9 and Variance = 9.25

2. First n natural numbers

Solution:-

We know that Mean = Sum of all observations/Number of observations

∴Mean, $\bar{x} = ((n(n + 1))2)/n$

$= (n + 1)/2$

and also, WKT Variance,

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \bar{x})^2$

By substituting the value of $\bar{x}$, we get

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \frac{n+1}{2}\right)^2$$

We know that $(a - b)^2 = a^2 - 2ab + b^2$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i)^2 - \frac{1}{n}\sum_{i=1}^{n}2x_i\left(\frac{n+1}{2}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

Substituting the summation values

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \frac{n+1}{n}\left[\frac{n(n+1)}{2}\right] + \frac{(n+1)^2}{4n} \times n$$

Multiplying and Computing

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

By taking LCM and simplifying, we get

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

By taking (n + 1) common from each term, we get

$$= (n+1)[\frac{4n+2-3n-3}{12}]$$

$$= \frac{(n+1)(n-1)}{12}$$

WKT, $(a + b)(a - b) = a^2 - b^2$

$\sigma^2 = (n^2 - 1)/12$

∴Mean $= (n + 1)/2$ and Variance $= (n^2 - 1)/12$

3. First 10 multiples of 3

Solution:-

First, we have to write the first 10 multiples of 3.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

We have,

Mean $= \bar{x} = \frac{\sum_{i=1}^{a} x_i}{n}$

Where, n = number of observation

$\sum_{i=1}^{a} x_i$ = sum of total observation

So, $\bar{x} = (3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30)/10$

$= 165/10$

$= 16.5$

Let us make the table of the data and append other columns after calculations.

Xi	Deviations from mean (xi – x̄)	(xi – x̄)2
3	3 – 16.5 = -13.5	182.25

6	6 – 16.5 = -10.5	110.25
9	9 – 16.5 = -7.5	56.25
12	12 – 16.5 = -4.5	20.25
15	15 – 16.5 = -1.5	2.25
18	18 – 16.5 = 1.5	2.25
21	21 – 16.5 = – 4.5	20.25
24	24 – 16.5 = 7.5	56.25
27	27 – 16.5 = 10.5	110.25
30	30 – 16.5 = 13.5	182.25
		742.5

Then, the Variance

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \bar{x})^2$

= (1/10) × 742.5

= 74.25

∴Mean = 16.5 and Variance = 74.25

4.

xi	6	10	14	18	24	28	30
fi	2	4	7	12	8	4	3

Solution:-

Let us make the table of the given data and append other columns after calculations.

Xi	fi	fixi	Deviations from mean (xi – x̄)	(xi – x̄)2	fi(xi – x̄)2
6	2	12	6 – 19 = 13	169	338
10	4	40	10 – 19 = -9	81	324
14	7	98	14 – 19 = -5	25	175
18	12	216	18 – 19 = -1	1	12
24	8	192	24 – 19 = 5	25	200
28	4	112	28 – 19 = 9	81	324
30	3	90	30 – 19 = 11	121	363
	N = 40	760			1736

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 760/40

= 19

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/40) × 1736

= 43.4

∴Mean = 19 and Variance = 43.4

5.

xi	92	93	97	98	102	104	109
fi	3	2	3	2	6	3	3

Solution:-
Let us make the table of the given data and append other columns after calculations.

Xi	fi	fixi	Deviations from mean (xi – x̄)	(xi – x̄)2	fi(xi – x̄)2
92	3	276	92 – 100 = -8	64	192
93	2	186	93 – 100 = -7	49	98
97	3	291	97 – 100 = -3	9	27
98	2	196	98 – 100 = -2	4	8
102	6	612	102 – 100 = 2	4	24
104	3	312	104 – 100 = 4	16	48
109	3	327	109 – 100 = 9	81	243
	N = 22	2200			640

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 2200/22

= 100

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/22) × 640

= 29.09

∴Mean = 100 and Variance = 29.09

6. Find the mean and standard deviation using short-cut method.

xi	60	61	62	63	64	65	66	67	68
fi	2	1	12	29	25	12	10	4	5

Solution:-

Let the assumed mean A = 64. Here, h = 1

We obtain the following table from the given data.

Xi	Frequency fi	Yi = (xi – A)/h	Yi2	fiyi	fiyi2
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12

66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
				0	286

Mean,

$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where A = 64, h = 1

So, $\bar{x} = 64 + ((0/100) \times 1)$

= 64 + 0

= 64

Then, the variance,

$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$

$\sigma^2 = (1^2/100^2)\ [100(286) - 0^2]$

= (1/10000) [28600 – 0]

= 28600/10000

= 2.86

Hence, standard deviation = $\sigma = \sqrt{2.886}$

= 1.691

∴ Mean = 64 and Standard Deviation = 1.691

Find the mean and variance for the following frequency distributions in Exercises 7 and 8.

7.

Classes	0 – 30	30 – 60	60 – 90	90 – 120	120 – 150	150 – 180	180 – 210
Frequencies	2	3	5	10	3	5	2

Solution:-

Let us make the table of the given data and append other columns after calculations.

Classes	Frequency fi	Midpoints xi	fixi	(xi – x̄)	(xi – x̄)2	fi(xi – x̄)2
0 – 30	2	15	30	-92	8464	16928
30 – 60	3	45	135	-62	3844	11532
60 – 90	5	75	375	-32	1024	5120
90 – 120	10	105	1050	-2	4	40
120 – 150	3	135	405	28	784	2352
150 – 180	5	165	825	58	3364	16820
180 – 210	2	195	390	88	7744	15488
	N = 30		3210			68280

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 3210/30

= 107

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/30) × 68280

= 2276

∴Mean = 107 and Variance = 2276

8.

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 –50
Frequencies	5	8	15	16	6

Solution:-
Let us make the table of the given data and append other columns after calculations.

Classes	Frequency fi	Midpoints xi	fixi	(xi – x̄)	(xi – x̄)2	fi(xi – x̄)2
0 – 10	5	5	25	-22	484	2420
10 – 20	8	15	120	-12	144	1152
20 – 30	15	25	375	-2	4	60
30 – 40	16	35	560	8	64	1024
40 –50	6	45	270	18	324	1944
	N = 50		1350			6600

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 1350/50

= 27

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/50) × 6600

= 132

∴Mean = 27 and Variance = 132

9. Find the mean, variance and standard deviation using the short-cut method.

Height in cms	70 – 75	75 – 80	80 – 85	85 – 90	90 – 95	95 – 100	100 – 105	105 – 110	110 – 115
Frequencies	3	4	7	7	15	9	6	6	3

Solution:-

Let the assumed mean, A = 92.5 and h = 5

Let us make the table of the given data and append other columns after calculations.

Height (class)	Number of children Frequency fi	Midpoint Xi	Yi = (xi – A)/h	Yi2	fiyi	fiyi2
70 – 75	3	72.5	-4	16	-12	48
75 – 80	4	77.5	-3	9	-12	36
80 – 85	7	82.5	-2	4	-14	28
85 – 90	7	87.5	-1	1	-7	7
90 – 95	15	92.5	0	0	0	0

95 – 100	9	97.5	1	1	9	9
100 – 105	6	102.5	2	4	12	24
105 – 110	6	107.5	3	9	18	54
110 – 115	3	112.5	4	16	12	48
	N = 60				6	254

Mean,

$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where, A = 92.5, h = 5

So, $\bar{x}$ = 92.5 + ((6/60) × 5)

= 92.5 + ½

= 92.5 + 0.5

= 93

Then, the Variance,

$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$

$\sigma^2 = (5^2/60^2)\,[60(254) - 6^2]$

= (1/144) [15240 – 36]

= 15204/144

= 1267/12

= 105.583

Hence, standard deviation = σ = √105.583

= 10.275

∴ Mean = 93, variance = 105.583 and Standard Deviation = 10.275

10. The diameters of circles (in mm) drawn in a design are given below.

Diameters	33 – 36	37 – 40	41 – 44	45 – 48	49 – 52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.
[Hint: First, make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 – 48.5, 48.5 – 52.5 and then proceed.]
Solution:-
Let the assumed mean, A = 42.5 and h = 4
Let us make the table of the given data and append other columns after calculations.

Height (class)	Number of children (Frequency fi)	Midpoint Xi	Yi = (xi – A)/h	Yi2	fiyi	fiyi2
32.5 – 36.5	15	34.5	-2	4	-30	60
36.5 – 40.5	17	38.5	-1	1	-17	17
40.5 – 44.5	21	42.5	0	0	0	0
44.5 – 48.5	22	46.5	1	1	22	22
48.5 – 52.5	25	50.5	2	4	50	100
	N = 100				25	199

Mean,

$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where, A = 42.5, h = 4

So, $\bar{x} = 42.5 + (25/100) \times 4$

$= 42.5 + 1$

$= 43.5$

Then, the Variance,

$$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$$

$\sigma^2 = (4^2/100^2)[100(199) – 25^2]$

= (1/625) [19900 – 625]

= 19275/625

= 771/25

= 30.84

Hence, standard deviation = σ = √30.84

= 5.553

∴ Mean = 43.5, variance = 30.84 and Standard Deviation = 5.553.

Exercise 15.3 Page: 375

1. From the data given below state which group is more variable, A or B?

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Solution:-

For comparing the variability or dispersion of two series, we calculate the coefficient of variance for each series. The series having greater C.V. is said to be more variable than the other. The series having lesser C.V. is said to be more consistent than the other.

Co-efficient of variation (C.V.) = (σ/ x̄) × 100

Where, σ = standard deviation, x̄ = mean

For Group A.

Marks	Group A fi	Midpoint Xi	Yi = (xi – A)/h	(Yi)2	fiyi	fi(yi)2

10 – 20	9	15	((15 – 45)/10) = -3	(-3)2 = 9	–27	81
20 – 30	17	25	((25 – 45)/10) = -2	(-2)2 = 4	–34	68
30 – 40	32	35	((35 – 45)/10) = –1	(-1)2 = 1	–32	32
40 – 50	33	45	((45 – 45)/10) = 0	02	0	0
50 – 60	40	55	((55 – 45)/10) = 1	12 = 1	40	40
60 – 70	10	65	((65 – 45)/10) = 2	22 = 4	20	40
70 – 80	9	75	((75 – 45)/10) = 3	32 = 9	27	81
Total	150				-6	342

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where A = 45,

and $y_i = (x_i – A)/h$

Here h = class size = 20 – 10

h = 10

So, $\bar{x}$ = 45 + ((-6/150) × 10)

= 45 – 0.4

= 44.6

Then, variance $\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$

$\sigma^2 = (10^2/150^2)\ [150(342) - (-6)^2]$
$= (100/22500)\ [51,300 - 36]$
$= (100/22500) \times 51264$
$= 227.84$
Hence, standard deviation $= \sigma = \sqrt{227.84}$
$= 15.09$
$\therefore$ C.V for group A $= (\sigma/\ \bar{x}) \times 100$
$= (15.09/44.6) \times 100$
$= 33.83$
Now, for group B.

Marks	Group B fi	Midpoint Xi	Yi = (xi – A)/h	(Yi)2	fiyi	fi(yi)2
10 – 20	10	15	((15 – 45)/10) = -3	(-3)2 = 9	– 30	90
20 – 30	20	25	((25 – 45)/10) = -2	(-2)2 = 4	– 40	80
30 – 40	30	35	((35 – 45)/10) = – 1	(-1)2 = 1	– 30	30
40 – 50	25	45	((45 – 45)/10) = 0	02	0	0
50 – 60	43	55	((55 – 45)/10) = 1	12 = 1	43	43
60 – 70	15	65	((65 – 45)/10) = 2	22 = 4	30	60

70 – 80	7	75	((75 – 45)/10) = 3	32 = 9	21	63
Total	150				-6	366

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

Where A = 45,
h = 10
So, x̄ = 45 + ((-6/150) × 10)
= 45 – 0.4
= 44.6

Then, variance $\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$

$\sigma^2 = (10^2/150^2)\ [150(366) - (-6)^2]$
= (100/22500) [54,900 – 36]
= (100/22500) × 54,864
= 243.84
Hence, standard deviation = σ = √243.84
= 15.61
∴ C.V for group B = (σ/ x̄) × 100
= (15.61/44.6) × 100
= 35
By comparing the C.V. of group A and group B.
C.V of Group B > C.V. of Group A
So, Group B is more variable.

2. From the prices of shares X and Y below, find out which is more stable in value.

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

Solution:-
From the given data,
Let us make the table of the given data and append other columns after calculations.

X (xi)	Y (yi)	Xi2	Yi2
35	108	1225	11664
54	107	2916	11449
52	105	2704	11025
53	105	2809	11025
56	106	8136	11236
58	107	3364	11449
52	104	2704	10816
50	103	2500	10609
51	104	2601	10816
49	101	2401	10201
Total = 510	1050	26360	110290

We have to calculate Mean for x,
Mean $\bar{x} = \sum x_i/n$
Where, n = number of terms
= 510/10
= 51

Then, Variance for x = $\frac{1}{n^2}\left[N\sum x_i^2 - \left(\sum x_i\right)^2\right]$
= $(1/10^2)[(10 \times 26360) - 510^2]$
= (1/100) (263600 – 260100)
= 3500/100
= 35

WKT Standard deviation = √variance
$= \sqrt{35}$
= 5.91
So, co-efficient of variation = $(\sigma/\bar{x}) \times 100$
$= (5.91/51) \times 100$
= 11.58
Now, we have to calculate Mean for y,
Mean $\bar{y} = \sum y_i/n$
Where, n = number of terms
= 1050/10
= 105

Then, Variance for y $= \frac{1}{n^2}\left[N\sum y_i^2 - \left(\sum y_i\right)^2\right]$
$= (1/10^2)[(10 \times 110290) - 1050^2]$
= (1/100) (1102900 – 1102500)
= 400/100
= 4
WKT Standard deviation = √variance
$= \sqrt{4}$
= 2
So, co-efficient of variation = $(\sigma/\bar{x}) \times 100$
$= (2/105) \times 100$
= 1.904
By comparing C.V. of X and Y,
C.V of X > C.V. of Y
So, Y is more stable than X.

3. An analysis of monthly wages paid to workers in two firms, A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No. of wages earners	586	648
Mean of monthly wages	Rs 5253	Rs 5253
Variance of the distribution of wages	100	121

(i) Which firm, A or B, pays a larger amount as monthly wages?
(ii) Which firm, A or B, shows greater variability in individual wages?
Solution:-
(i) From the given table,
Mean monthly wages of firm A = Rs 5253
and Number of wage earners = 586
Then,
Total amount paid = 586 × 5253
= Rs 3078258
Mean monthly wages of firm B = Rs 5253
Number of wage earners = 648
Then,
Total amount paid = 648 × 5253
= Rs 34,03,944
So, firm B pays larger amount as monthly wages.
(ii) Variance of firm A = 100
We know that, standard deviation (σ)= $\sqrt{100}$
=10
Variance of firm B = 121
Then,
Standard deviation (σ)=$\sqrt{121}$
=11
Hence, the standard deviation is more in case of Firm B. That means, in firm B, there is greater variability in individual wages.
4. The following is the record of goals scored by team A in a football session:

No. of goals scored	0	1	2	3	4
No. of matches	1	9	7	5	3

For team B, the mean number of goals scored per match was 2, with a standard deviation 1.25 goals. Find which team may be considered more consistent?
Solution:-
From the given data,
Let us make the table of the given data and append other columns after calculations.

Number of goals scored xi	Number of matches fi	fixi	Xi2	fixi2
0	1	0	0	0
1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
Total	25	50		130

First we have to calculate Mean for Team A,

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{50}{25} = 2$

Then,

Variance $= \frac{1}{N^2}\left[N\sum f_i x_i^2 - (\sum f_i x_i)^2\right]$

$= \frac{1}{25^2}[25 \times 130 - 2500] = \frac{750}{625} = 1.2$

We know that, Standard deviation σ = √variance = √1.2 = 1.09

Hence co-efficient of variation of team A,

$C.V._A = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.09}{2} \times 100 = 54.5$

For team B

Given, $\bar{x} = 2$

Standard deviation σ = 1.25

So, co-efficient of variation of team B,

$\Rightarrow C.V._B = \frac{1.25}{2} \times 100 = 62.5$

C.V. of firm B is greater.

∴ Team A is more consistent.

5. The sum and sum of squares corresponding to length x (in cm) and weight y (in gm) of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212, \quad \sum_{i=1}^{50} x_i^2 = 902.8, \quad \sum_{i=1}^{50} y_i = 261, \quad \sum_{i=1}^{50} y_i^2 = 1457.6$$

Which is more varying, the length or weight?

Solution:-

First, we have to calculate Mean for Length x.

$$\text{Mean} = \bar{x} = \frac{\sum x_i}{n} = \frac{212}{50} = 4.24$$

Then,

$$\text{Variance} = \frac{1}{N^2}\left[N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2\right]$$

$= (1/50^2)\ [(50 \times 902.8) - 212^2]$

$= (1/2500)\ (45140 - 44944)$

$= 196/2500$

$= 0.0784$

We know that, Standard deviation σ = √variance

$= \sqrt{0.0784}$

$= 0.28$

Hence co-efficient of variation of team A,

$$C.V._x = \frac{\sigma}{\bar{x}} \times 100 = \frac{0.28}{4.24} \times 100 = 6.603$$

Now we have to calculate mean of Weight y

$\bar{y} = \sum y_i / n$

= 261/50

= 5.22

Then,

Variance = $(1/N^2) [(N\sum f_i y_i^2) - (\sum f_i y_i)^2]$

= $(1/50^2) [(50 \times 1457.6) - 261^2]$

= (1/2500) (72880 - 68121)

= 4759/2500

= 1.9036

We know that, Standard deviation σ = √variance

= √1.9036

= 1.37

So, co-efficient of variation of team B,

$$C.V._Y = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.37}{5.22} \times 100 = 26.24$$

Since C.V. of firm weight y is greater

∴ Weight is more varying.

Miscellaneous Exercise Page: 380

1. The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:-

Form the question it is given that,

Variance of eight observations are 9 and 9.25.

There are six observations given 6, 7, 10, 12, 12, and 13

Let us assume the remaining two observations to be x and y respectively such that,

Observations: 6, 7, 10, 12, 12, 13, x, y.

We have to calculate the mean of given observations,

$\therefore$ Mean, $\overline{x} = \frac{6+7+10+12+12+13+x+y}{8}$

$$9 = \frac{6+7+10+12+12+13+x+y}{8}$$

$60 + x + y = 72$

$x + y = 12$ ... [we call it as equation (i)]

$$\text{Now, Variance } = \frac{1}{n}\sum_{i=1}^{8}(x_i - x)^2$$

$$9.25 = \frac{1}{8}[(-3)^2 + (-2)^2 + 1^2 + 3^2 + 4^2 + x^2 + y^2 - 18(x+y) + 2 \times 9^2]$$

By using equation (i) substitute 12 instead of (x + y)

$$9.25 = \frac{1}{8}[9 + 4 + 1 + 9 + 9 + 16 + x^2 + y^2 - 18 \times 12 + 162]$$

$$9.25 = \frac{1}{8}[48 + x^2 + y^2 - 216 + 162]$$

$$9.25 = \frac{1}{8}[x^2 + y^2 - 6]$$

$x^2 + y^2 = 80$ ... [we call it as equation (ii)]

So, from equation (i) we have:

$x^2 + y^2 + 2xy = 144$ (iii)

Thus, from (ii) and (iii), we have

$2xy = 64$ (iv)

Now by subtracting (iv) from (ii), we get:

$x^2 + y^2 - 2xy = 80 - 64$

$x - y = \pm 4$ (v)

Hence, from equation (i) and (v) we have:

When $x - y = 4$

Then, $x = 8$ and $y = 4$

And, when $x - y = -4$

Then, $x = 4$ and $y = 8$

∴ The remaining observations are 4 and 8

2. The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14. Find the remaining two observations.

Solution:-

Form the question it is given that,

Variance of seven observations are 8 and 16.

There are six observations given 2, 4, 10, 12, and 14

Let us assume the remaining two observations to be x and y respectively such that,

Observations: 2, 4, 10, 12, 14, x, y.

We have to calculate the mean of given observations,

$$\therefore \text{Mean}, \bar{x} = \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8$$

$x + y = 14$ ... [we call it as equation (i)]

In the question it is given that,

Variance = 16

We know that,

$$\text{Variance} = \frac{1}{n}\sum_{i=1}^{n}(x_1 - \bar{x})^2$$

$$16 = \frac{1}{7}[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 2 \times 8\,(x + y) + 2 \times (8)^2]$$

By using equation (i) substitute 14 instead of (x + y)

$$16 = \frac{1}{7}\,[36 + 16 + 4 + 16 + 36 + x^2 + y^2 - 16\,(14) + 2\,(64)]$$

$$16 = \frac{1}{7}\,[12 + x^2 + y^2]$$

$x^2 + y^2 = 112 - 12$

$x^2 + y^2 = 100$ ... [we call it as equation (ii)]

So, from equation (i) we have:

$x^2 + y^2 + 2xy = 196$ ... [we call it as equation (iii)]

Thus, from equation (ii) and (iii) we have:

$2xy = 196 – 100$

$2xy = 96$ (iv)

Now subtracting equation (iv) from (ii),

We get:

$x^2 + y^2 – 2xy = 100 – 96$

$(x – y)^2 = 4$

$x – y = \pm 2$ (v)

Hence, from equation (i) and (v) we have:

When $x – y = 2$ then $x = 8$ and $y = 6$

And, when $x – y = - 2$ then $x = 6$ and $y = 8$

∴ The remaining observations are 6 and 8

3. The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution:-

From the question it is given that,

Mean of six observations = 8

Standard deviation of six observations = 4

Let us assume the observations be x_1, x_2, x_3, x_4, x_5 and x_6

So, mean of assumed observations,

$$\therefore \text{Mean } \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8$$

Then, as per the question if each observation is multiplied by 3 and the resulting observations are y_i then, we have:

$y_i = 3x_i$

Hence, $x_i = \frac{1}{3} y_i$ (For i = 1 to 6)

$$\therefore \text{New mean, } \bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$$

$$= \frac{3\,(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6}$$

$= 3 \times 8$

$= 24$

We know that,

Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{6}(x_i - \bar{x})^2}$

By squaring on both the sides

$$\therefore (4)^2 = \frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2$$

$$\sum_{i=1}^{6}(x_i - \bar{x})^2 = 96 \text{ (ii)}$$

Hence, from (i) and (ii) we have:

$\bar{y} = 3\bar{x}$

$\bar{x} = \frac{1}{3}\bar{y}$

Now, by substituting the values of x_i and $\bar{x}$ in (ii) we have:

$$\sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}\bar{y}\right)^2 = 96$$

Thus, $\sum_{i=1}^{6}(y_i - \bar{y})^2 = 864$

So, the variance of new observation = (1/6) × 864

= 144

Therefore, standard deviation of new observation = $\sqrt{144}$

= 12

4. Given that $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, \ldots, x_n$. Prove that the mean and variance of the observations $ax_1, ax_2, ax_3, \ldots, ax_n$ are $a\bar{x}$ and $a^2\sigma^2$, respectively, $(a \neq 0)$.

Solution:-

From the question, it is given that, n observations are $x_1, x_2, \ldots x_n$

Mean of the n observation $= \bar{x}$

Variance of the n observation $= \sigma^2$

As we know,

Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} y_1(x_i - \bar{x})^2$ ... [equation (i)]

As per the condition given in the question, if each of the observation is being multiplied by 'a' and the new observation are y_i the, we have:

$y_i = ax_i$

Thus, $x_i = \frac{1}{a} y_i$

$$\therefore \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} ax_i$$

$$\bar{y} = \frac{a}{n}\sum_{i=1}^{n} x_i$$

$$\bar{y} = a\bar{x}$$

Therefore, mean of the observations $ax_1, ax_2, \ldots ax_n$ is a $\bar{x}$

Now, by substituting the values of x_i and $\bar{x}$ in equation(i), we get:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{a}y_1 - \frac{1}{a}\bar{y}\right)^2$$

$$a^2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(y_1 - \bar{y})^2$$

$\therefore$ the variance of the given observations $ax_1, ax_2, \ldots ax_n$ is $a^2\sigma^2$

5. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases: (i) If wrong item is omitted. (ii) If it is replaced by 12

Solution:-

(i) If the wrong item is omitted,

From the question, it is given that

The number of observations, i.e., n = 20

The incorrect mean = 20

The incorrect standard deviation = 2

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{20} X_1$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} X_1$$

$$\sum_{i=1}^{20} X_1 = 200$$

By the calculation the incorrect sum of observations = 200

Hence, correct sum of observations = 200 – 8

= 192

Therefore the correct mean = correct sum/19

= 192/19

= 10.1

We know that, Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X_{1}\right)^2}$

$$2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1^2 - (\overline{X})^2}$$

$$4 = \frac{1}{20}\text{ Incorrect }\sum_{i=1}^{n} X_1^2 - 100$$

$$\text{Incorrect }\sum_{i=1}^{n} X_1^2 = 2080$$

Therefore, correct $\sum_{i=1}^{n} X_1^2 =$ Incorrect $\sum_{i=1}^{n} X_1^2 - (8)^2$

= 2080 – 64

= 2016

Finally we came to calculate correct standard deviation,

Hence, Correct standard deviation $= \sqrt{\frac{\text{Correct } \sum X_1^2}{n} - (\text{Correct Mean})^2}$

$$= \sqrt{\frac{2016}{19} - (10.1)^2}$$

$$= \sqrt{1061.1 - 102.1}$$

= 2.02

(ii) If it is replaced by 12,
From the question, it is given that
The number of incorrect sum observations, i.e., n = 200
The correct sum of observations n = 200 – 8 + 12
n = 204
Then, correct mean = correct sum/20
= 204/20
= 10.2

Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X_{\;1}\right)^2}$

$$\therefore 2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1^2 - (\bar{X})^2}$$

$4 = \frac{1}{20}$ Incorrect $\sum_{i=1}^{n} X_1^2 - 100$

Incorrect $\sum_{i=1}^{n} X_1^2 = 2080$

Thus, correct $\sum_{i=1}^{n} X_1^2 =$ Incorrect $\sum_{i=1}^{n} X_1^2 - (8)^2 + (12)^2$

$= 2080 - 64 + 144$

$= 2160$

Hence, Correct standard deviation $= \sqrt{\frac{\text{Correct } \sum X_1^2}{n} - (\text{Correct Mean})^2}$

$= \sqrt{\frac{2160}{20} - (10.2)^2}$

$= \sqrt{108 - 104.04}$

$= \sqrt{3.96}$

$= 1.98$

6. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry, are given below.

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9

Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks, and which shows the lowest?
Solution:-
From the question, it is given that
Mean of Mathematics = 42
Standard deviation of Mathematics = 12
Mean of Physics = 32
Standard deviation of physics = 15
Mean of Chemistry = 40.9
Standard deviation of chemistry = 20
As we know,

$$\text{Coefficient of variation (C.V)} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

Then,

C.V. in Mathematics = (12/42) × 100

= 28.57

C.V. in Mathematics = (15/32) × 100

= 46.87

C.V. in Mathematics = (20/40.9) × 100

= 48.89

Hence, subject with highest variability in marks is chemistry as subject with the greater C.V is more variable than others

7. The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on, it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.
Solution:-
From the question, it is given that
The total number of observations (n) = 100
Incorrect mean, $(\bar{x}) = 20$
And, Incorrect standard deviation $(\sigma) = 3$

$$\therefore 20 = \frac{1}{100}\sum_{i=1}^{100} X_1$$

By cross multiplication, we get

$$\sum_{i=1}^{100} X_1 = 20 \times 100$$

$$\sum_{i=1}^{100} X_1 = 2000$$

Hence, incorrect sum of observations is 2000

Now, correct sum of observations = 2000 – 21 – 21 – 18

= 2000 – 60

= 1940

Therefore correct Mean = Correct sum/(100 – 3)

= 1940/97

= 20

We know that, Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X\right)^2}$

$$3 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1^2 - (\overline{X})^2}$$

$$3 = \sqrt{\frac{1}{100} \times \text{Incorrect} \sum X_1^2 - (20)^2}$$

$$\text{Incorrect} \sum X_1^2 = 100\,(9 + 400)$$

$$\text{Incorrect} \sum X_1^2 = 40900$$

$$\text{Correct} \sum_{i=1}^{n} X_1^2 = \text{Incorrect} \sum_{i=1}^{n} X_1^2 - (21)^2 - (21)^2 - (18)^2$$

$$= 40900 - 441 - 441 - 324$$

$$= 40900 - 1206$$

$$= 39694$$

$$\text{Hence, correct standard deviation} = \sqrt{\frac{\text{Correct} \sum X_1^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{39694}{97} - (20)^2}$$

$$= \sqrt{409.216 - 400}$$

$$= 3.036$$

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